

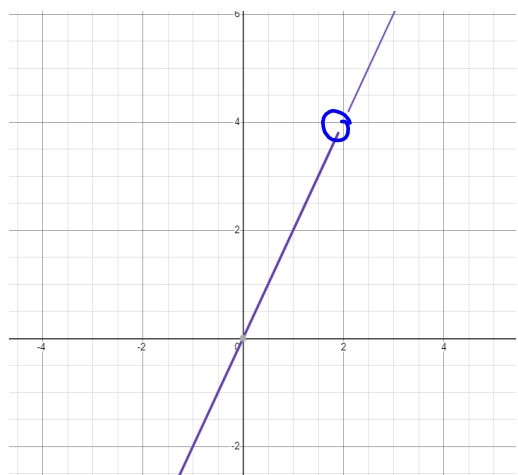


LIMITS - GRAPHICALLY

The limit of a function is a very important concept in calculus.

Ex.1

$$f(x) = 2x \quad x \neq 2$$



x	$f(x)$
1.9	3.8
1.99	3.98
1.999	3.998
2.0	4 ←
2.001	4.002
2.01	4.02
2.1	4.2

$$\lim_{x \rightarrow 2^-} f(x) = 4$$

$$\lim_{x \rightarrow 2^+} f(x) = 4$$

$$\left. \begin{array}{l} \lim_{x \rightarrow 2^-} f(x) = 4 \\ \lim_{x \rightarrow 2^+} f(x) = 4 \end{array} \right\} \rightarrow \lim_{x \rightarrow 2} f(x) = \underline{4}$$

$$f(2) \text{ DNE}$$

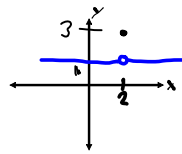
Does not exist

Let's look at limits graphically.

Ex.2

Graph $\frac{x-2}{x-2} = 1 \quad x \neq 2$

$$f(x) = \begin{cases} \frac{x-2}{x-2} & x \neq 2 \\ 3 & x = 2 \end{cases}$$



$$\lim_{x \rightarrow 2^-} f(x) = 1 \quad \lim_{x \rightarrow 2^+} f(x) = 1 \quad \lim_{x \rightarrow 2} f(x) = 1 \quad f(2) = 3$$

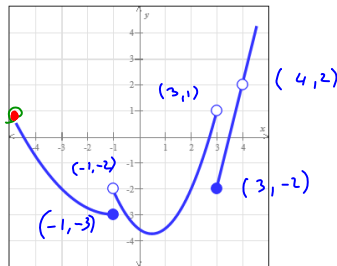
A limit exists iff the left hand limit and the right hand limit exist and they are both equal.

$$\lim_{x \rightarrow a} f(x) \text{ IFF } \lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x)$$

Ex.3

The function f(x) is defined below. Find the following

- a) $\lim_{x \rightarrow -1^+} f(x)$
- b) $\lim_{x \rightarrow -1^-} f(x)$
- c) $\lim_{x \rightarrow -1} f(x)$



a) $\lim_{x \rightarrow -1^+} f(x) = -2$

b) $\lim_{x \rightarrow -1^-} f(x) = -3$

c) $\lim_{x \rightarrow -1} f(x) = \text{DNE}$

$\lim_{x \rightarrow -5^-} f(x) = 1$

$\lim_{x \rightarrow -5^+} f(x) = 1$

$\lim_{x \rightarrow -5} f(x) = \text{DNE}$

TRY

Using the above find

- a) $\lim_{x \rightarrow 3^+} f(x) = -2$
- b) $\lim_{x \rightarrow 3^-} f(x) = 1$
- c) $\lim_{x \rightarrow 3} f(x) = \text{DNE}$

- d) $\lim_{x \rightarrow 4^+} f(x) = 2$
- e) $\lim_{x \rightarrow 4^-} f(x) = 2$
- f) $\lim_{x \rightarrow 4} f(x) = 2$

$f(?) = \text{DNE}$